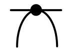
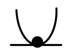
Analyzing functions - practice

- **Local (Relative) Max and Local Min:** where
  $f'(x) = 0$ and $f''(x) < 0$ for local max  (slope of tangent line = 0, concave down)
  $f'(x) = 0$ and $f''(x) > 0$ for local min  (slope of tangent line = 0, concave up)
  $f'(x)$ does not exist but $f(x)$ does
- **Global Max and Global Min:** The absolute highest and lowest points of the function including the end points.

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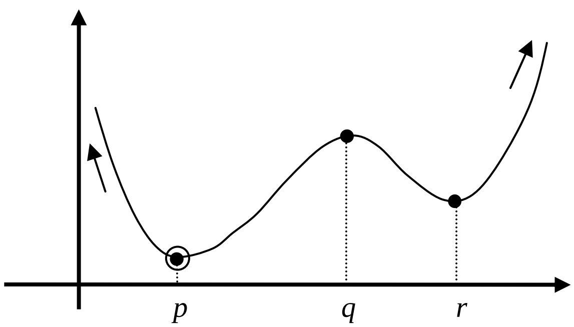
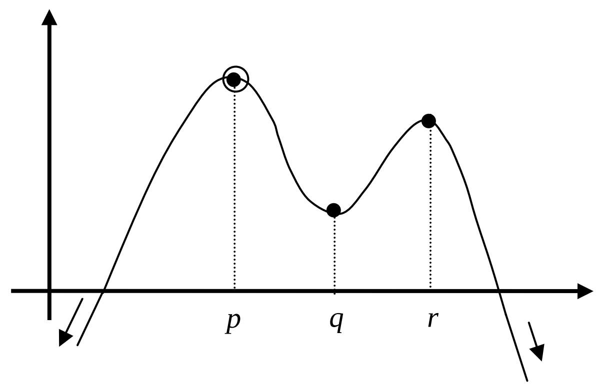
a) Open Interval, No End Points (entire real line):





- Local Maximum at: ***q***
- Local Minimum at: ***p*** and ***r***
- **No** Global (*Absolute*) Maximum
- Global (*Absolute*) Minimum at: ***p***

- Local Maximum at: ***p*** and ***r***
- Local Minimum at: ***q***
- Global (*Absolute*) Maximum at: ***p***
- **No** Global (*Absolute*) Minimum

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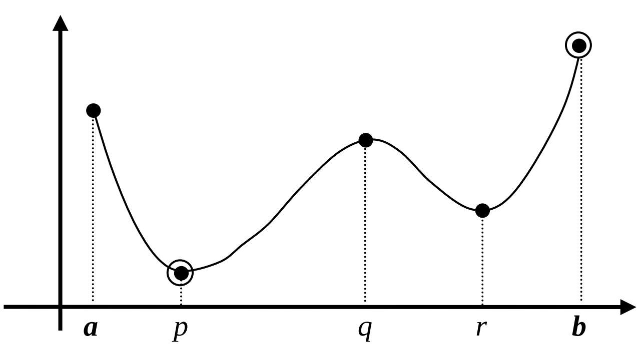
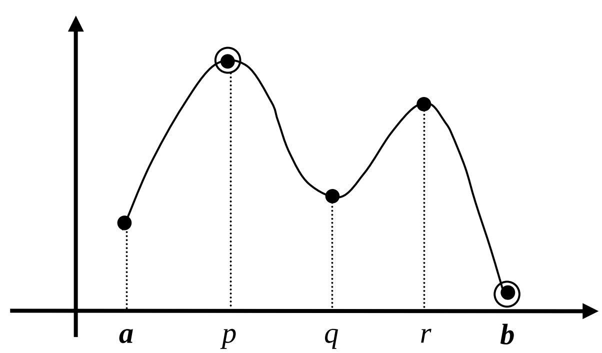
b) Closed Interval, With End Points such as $a \le x \le b$:





- Local Maximum at: ***a, q and b***
- Local Minimum at: ***p*** and ***r***
- Global (*Absolute*) Maximum at: ***b***
- Global (*Absolute*) Minimum at: ***p***

- Local Maximum at: ***p*** and ***r***
- Local Minimum at: ***a***, ***b***, and ***q***
- Global (*Absolute*) Maximum at: ***p***
- Global (*Absolute*) Minimum at: ***b***

The following example is similar to problems 18, 19 and 20 on page 186.

Example 1: **For the function** $f(x) = -x^3 + 3x^2 - 4$; $(-1.5 \leq x \leq 3)$

a) Find the f' and f''.

b) Find the critical points.

c) Find the inflection points.

d) Evaluate f at its critical points and the endpoints of the given interval. Identify local and global maxima and minima in the interval.

e) Graph f

The following examples are similar to the final exam style (keep your work to review for final exam).

Example 2: **For the function** $f(x) = x^3 - 3x^2 + 6$; $(-1.1 \leq x \leq 2.5)$

a) Find the f' and f''.

b) Find the critical points.

c) Find the inflection points.

d) Use 1st or 2nd derivative test to classify the critical points as local max or local min.

e) Find any global max or global min

f) Sketch a graph of the function.

Example 3: **For the function** $f(x) = 2x^3 - 6x + 2$; $(-1.5 \leq x \leq 2)$

a) Find the f' and f''.

b) Find the critical points.

c) Find the inflection points.

d) Use 1st or 2nd derivative test to classify the critical points as local max or local min.

e) Find any global max or global min

f) Sketch a graph of the function.

Answers for Example 3:

Critical points at $x = -1$ and $x = 1$. Inflection point at (0, 2)

Local Max at (-1.5, 4.25) ; Global Min at (1, -2) ; Global Max at (-1, 6) and (2, 6)

Example 1 Solution: $f(x) = -x^3 + 3x^2 - 4$; $(-1.5 \le x \le 3)$

a) $f'(x) = -3x^2 - 6x$; $f''(x) = -6x - 6$

b) Critical points where $f'(x) = 0$, then

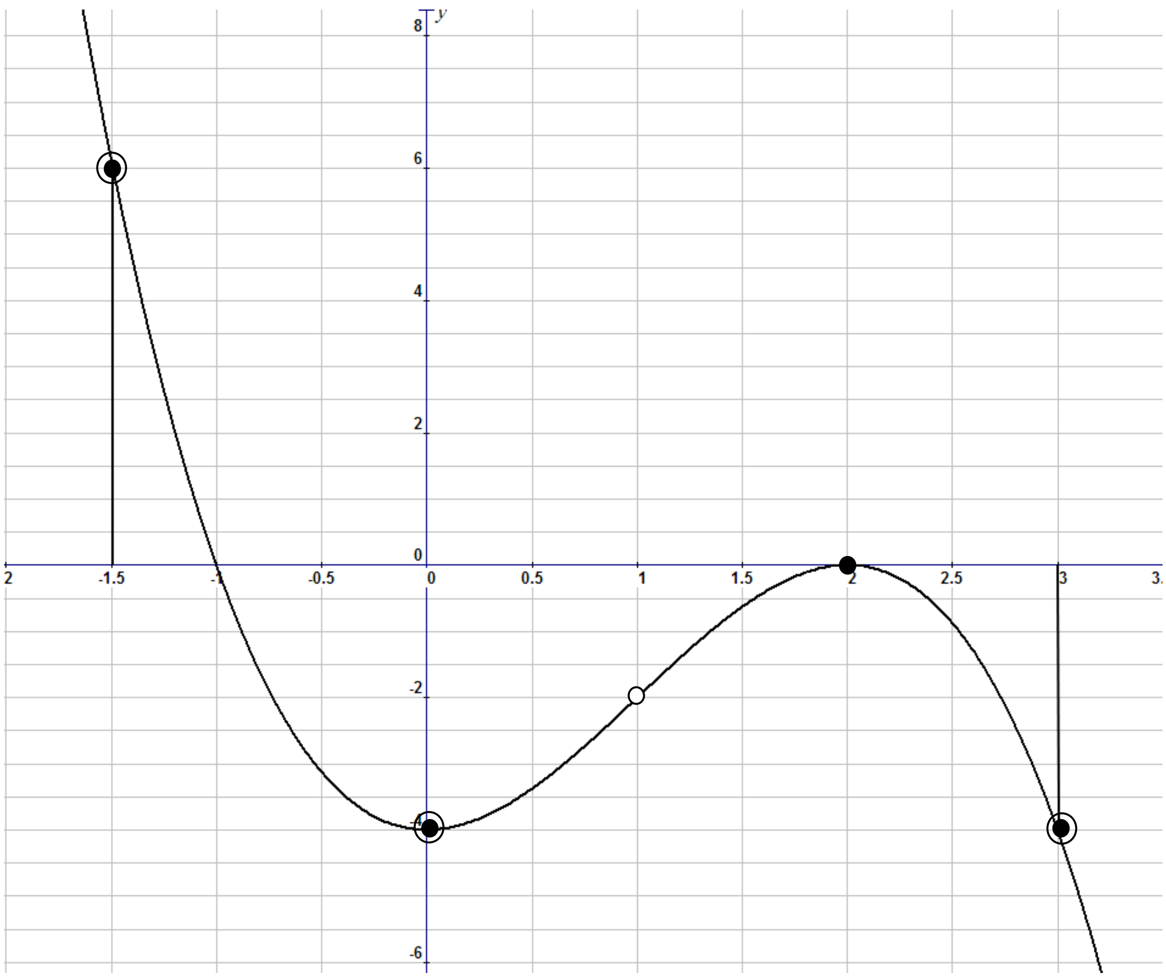
$-3x^2 - 6x = 0$ or $-3x(x-2) = 0$ ➔ $x = 0$ and $x = 2$

c) Inflection points where $f''(x) = 0$, then

$-6x - 6 = 0$ or $-6(x-1) = 0$ ➔ $x = 1$, $y = -2$

(Substitute x = 1 in the original function of $f(x) = -x^3 + 3x^2 - 4$ *to get y = -2).*

d) $x = 0$	➔	$f(0) = -4$	Global Min at (0,-4)
$x = 2$	➔	$f(2) = 0$	Local Max at (2, 0)
$x = -1.5$ (end point)	➔	$f(-1.5) = 6.125$	Global Max at (-1.5, 6.125)
$x = 3$ (end point)	➔	$f(3) = -4$	Global Min at (3, -4)



<u>Example 2 Solution</u>: $f(x) = x^3 - 3x^2 + 6$; $(-1.1 \le x \le 2.5)$

a) $f'(x) = 3x^2 - 6x$; $f''(x) = 6x - 6$

b) Critical points where $f'(x) = 0$, then

$3x^2 - 6x = 0$ or $3x(x-2) = 0$ ➔ $x = 0$ and $x = 2$

c) Inflection points where $f''(x) = 0$, then

$6x - 6 = 0$ or $6(x-1) = 0$ ➔ $x = 1, y = 4$

(Substitute $x = 1$ in the original function of $f(x) = x^3 - 3x^2 + 6$ to get $y = 4$).

d) Local Max and Local Min at the critical points of $x = 0$ and $x = 2$. Substitute each point in the second derivative and check the sign of the second derivative:

$f''(0) = 6(0) - 0 = -6 < 0$; concave down Local Max at $x = 0$

$f''(2) = 6(2) - 2 = 6 > 0$; concave up Local Min at $x = 2$

e) Use End Points and critical points to check Global Max and Global Min:

$x = 2$	➔	$f(2) = 2$	Local Min at (2,2)
$x = 0$	➔	$f(0) = 6$	Global Max at (0, 6)
$x = -1.1$ (end point)	➔	$f(-1.1) = 1.04$	Global Min at (-1.1, 1.04)
$x = 2.5$ (end point)	➔	$f(2.5) = 2.875$	Local Max at (2.5, 2.875)